Week 5 Worksheet Tuesday

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Instructions. Discuss with your group mates and do the following problems. You are not expected to finish all the problems. :)

1. $\lim_{t\to 0}\frac{t}{\sin t}$

$= 1$

2. $\lim_{x\to 0} x\sin\left(\frac{1}{x}\right)$

sandwich thm.

$-1 \le \sin(\frac{1}{x}) \le 1$

$-|x| \le x\sin(\frac{1}{x}) \le |x|$

$\to 0$ $\quad\swarrow$ as $x\to 0$

$\Rightarrow \lim_{x\to 0} x\sin(\frac{1}{x}) = 0$

2'. $\lim_{x\to+\infty} x\sin(\frac{1}{x})$

$= \lim_{x\to+\infty} \frac{\sin(\frac{1}{x})}{\frac{1}{x}} \cdot x\cdot\frac{1}{x}$

$= 1$

3. $\lim_{x\to 0}\frac{x}{2\tan x}$

$= \lim_{x\to 0} \frac{x\cos x}{2\sin x}$

$= \frac{1}{2}\cdot 1\cdot 1$

$= \frac{1}{2}$

4. $\lim_{\theta\to 0}\frac{\sin(5\theta)}{3\theta}$ *(similar to 2014 and 2015 exams)*

$= \lim_{\theta\to 0} \frac{\sin(5\theta)}{5\theta}\ \frac{5\theta}{3\theta}$

$= 1\cdot\frac{5}{3}$

$= \frac{5}{3}$

5. $\lim_{x\to 0}\frac{\sin(3x^2)}{\sin^2(2x)}$

$= \lim_{x\to 0} \frac{\sin(3x^2)}{3x^2}\ \frac{2x}{\sin(2x)}\ \frac{2x}{\sin(2x)}\ \frac{3x^2}{(2x)^2}$

$= 1\cdot 1\cdot 1\cdot\frac{3}{4}$

$= \frac{3}{4}$

6. $\lim_{x\to 0}\frac{\tan(4x)}{\sin(3x)}$

$= \lim_{x\to 0} \frac{\sin(4x)}{\cos(4x)}\cdot\frac{1}{\sin(3x)}$

$= \lim_{x\to 0} \frac{\sin(4x)}{4x}\ \frac{1}{\cos(4x)}\ \frac{3x}{\sin(3x)}\cdot\frac{4x}{3x}$

$= 1\cdot 1\cdot 1\cdot\frac{4}{3}$

$= \frac{4}{3}$

7. $\lim_{x\to -2}\frac{\sin(3x+6)}{x(x+2)}$ *(inspired by 2014 fall)*

$= \lim_{x\to -2} \frac{\sin(3x+6)}{3x+6}\ \frac{3x+6}{x(x+2)}$

$= \lim_{x\to -2} \frac{\sin(3x+6)}{3x+6}\ \frac{3(x+2)}{x(x+2)}$

$= 1\cdot\frac{3}{-2}$

$= -\frac{3}{2}$

8. $\lim_{x\to 0}\frac{x+x\cos x}{\sin x\cos x}$ *(2015 fall)*

Way 1:

$= \lim_{x\to 0} \frac{x}{\sin x}\ \frac{x+x\cos x}{x\cos x}$

$= \lim_{x\to 0} \frac{x}{\sin x}\ \frac{1+\cos x}{\cos x}$

$= 1\cdot\frac{1+1}{1}$

$= 2$

Way 2:

$= \lim_{x\to 0} \frac{x}{\sin x\cos x} + \frac{x\cos x}{\sin x\cos x}$

$= 1+1 = 2$

9. $\lim_{x\to 0}\frac{x^2+2x-\sin x}{3x}$ *(Similar to hw)*

$=\lim_{x\to 0}\frac{x^2+2x}{3x}-\frac{\sin x}{3x}$

$=\lim_{x\to 0}\frac{x+2}{3}-\frac{1}{3}\frac{\sin x}{x}$

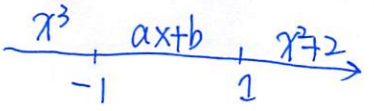
$=\frac{2}{3}-\frac{1}{3}$

$=\frac{1}{3}$

10. A function is defined by

$$f(x)=\begin{cases} x^3 & for\ x<-1 \\ ax+b & for\ -1\le x\le 1 \\ x^2+2 & for\ x\ge 1 \end{cases}$$

Determine the values of a and b, such that $f(x)$ is continuous.



Because left limit = right limit, we have

$$\begin{cases} -a+b=-1 \\ a+b=3 \end{cases} \Rightarrow \begin{cases} a=2 \\ b=1 \end{cases}$$

11. A function is defined by

$$f(x)=\begin{cases} \frac{x+|x|}{x} & if\ x\neq 0 \\ a & if\ x=0 \end{cases}$$

Is there a value of a, such that $f(x)$ is continuous? If so, determine a; if not, explain why.

Can not find a.

because $\lim_{x\to 0}\frac{x+|x|}{x}$ does not exist!

This is a removable discontinuity.